Precoding for Multiple-Speed Data Transmission

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In certain applications, because of noise, compatibility, or other considerations, it is desirable that a data transmission system have the flexibility to operate at multiple speeds. In this paper, a precoding scheme for multiple-speed digital or analog data transmission is presented. The scheme has a flexibility which allows the data rate and overall channel characteristics to be changed simultaneously by simply changing the data format and some resistive elements. There is no change in the filters, the equalization, the transmitter signaling interval, or the receiver sampling time. By using partial response channels, a number of commonly used data rates are easily obtained, using a physically realizable precoder and correlator. With correct timing and the use of orthonormal siginals, the signal-to-noise ratio is maximized at each data rate for bandlimited white noise under the constraints of fixed line signal power and no intersymbol interference. Timing error is considered in a two-speed transmission scheme, and the selection of a precoding matrix using eye opening as the criterion is studied. This study clearly demonstrates the advantage of changing the overall channel characteristics when changing the data rate. Eye openings obtained are equal to or larger than those of two conventional schemes transmitting at the same data rates.

I. INTRODUCTION

In conventional pulse amplitude modulation (PAM) data transmission systems (digital or analog), the signal at the receiver input takes the form

$$s(t) = \sum_{k=1}^{N} a_k f(t - kT_0), \tag{1}$$

where $\{a_k\}$ are the information symbols, T_0 is the signaling interval, and the signals $f(t - kT_0)$, $k = 1, \cdots, N$, are time translates of each other. It is well known[1] that in order for these systems to meet the criterion "Maximize the signal-to-noise ratio in the presence of band-limited white noise under the constraints of fixed line signal power and no intersymbol interference," the signals should be designed so that the overall channel characteristics are in the Nyquist I class and the overall amplitude characteristics are divided equally between the transmitting and the receiving side. Such a signal design scheme (hereafter referred to as Scheme I) is popular and is used even if the system designer is aware that the channel noise may not be white over the frequency band of interest. This is because the practical determination of the noise statistics and the realization of the corresponding optimum filters for a general communication complex are nearly impossible. A block diagram of Scheme I is shown in Fig. 1.

In this paper, a precoding signaling scheme (Scheme II) is presented for multiple-speed analog or digital data transmission. Scheme II also meets the signal-to-noise ratio criterion above. The very distinctive difference between Schemes I and II is that in I the signals $f(t - kT_0)$ are time translates, but in II the signals are not necessarily so. This property allows the data rate and overall channel characteristics (such as represented by the eye opening) of Scheme II to be changed simultaneously without changing the filters, the equalization, the signaling interval at the transmitter, or the sampling time at the receiver.

In Scheme II, a sequence of information symbols is divided into blocks and the blocks are transmitted sequentially. For clarity, we first consider in Section II the transmission of a single block at a fixed data rate and the precoder and the receiver structure. Multiple block multispeed transmission and the use of partial response channels are considered in Section III. A two-speed transmission scheme, sampling time error, and eye patterns are considered in Section IV.

II. TRANSMISSION OF A SINGLE BLOCK AT A FIXED DATA RATE

A block diagram of Scheme II is shown in Fig. 2. The quantities $H(j\omega)$ and $h(t)$ are, respectively, the transfer function and the impulse

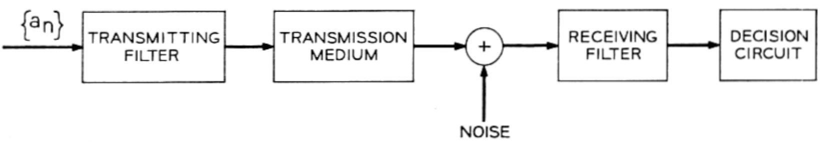

Fig. 1 — Block diagram of Scheme I.

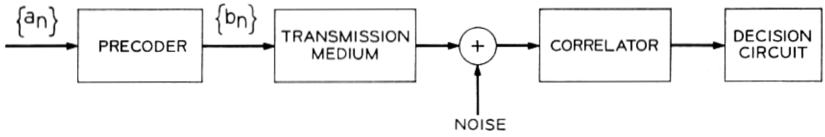

Fig. 2 — Block diagram of Scheme II.

response of the transmission medium. We shall consider $H(j\omega)$ to be bandlimited, and

$$\begin{aligned} H(j\omega) &\neq 0, \qquad |\omega| \leqq 2\pi f_c \\ &= 0, \qquad \text{otherwise.} \end{aligned} \tag{2}$$

The time interval

$$T = \frac{1}{2f_c} \quad \text{seconds} \tag{3}$$

is the Nyquist interval.

Consider the transmission of a block of symbols $a_1, \cdots, a_N$. Each symbol can be an m-ary digit ($m \geqq 2$) or a real number. The precoder converts $a_1, \cdots, a_N$ into a sequence of numbers $b_1, \cdots, b_N$, and the number b_k, $k = 1, \cdots, N$, is transmitted at $t = kT$. This produces a signal at the input to the receiver given by

$$s(t) = \sum_{k=1}^{N} b_k h(t - kT). \tag{4}$$

From (2), the impulse responses $h(t - kT)$ are infinitely linearly independent, i.e.,

$$\sum_{k=1}^{N} b_k h(t - kT) = 0 \quad \text{for all } t \Rightarrow b_k = 0 \quad \text{for all } k, \tag{5}$$

where N can approach infinity. Equation (5) can be proven by noting that the equality

$$\sum_{k=1}^{N} b_k h(t - kT) = 0 \quad \text{for all } t$$

and (2) together imply that

$$\sum_{k=1}^{N} b_k e^{-j\omega kT} = 0 \quad \text{for} \quad |\omega| \leqq 2\pi f_c . \tag{6}$$

Equation (6) then implies that $b_k = 0$ for all k.

As is well known, a bandlimited signal, say $g(t)$, can be represented by its time samples. The vector whose elements are the time samples of $g(t)$ will be referred to as the time sample vector of $g(t)$. For convenience, we shall use time sample vectors in discussing the precoder and receiver structures, and use the signals themselves in analyzing the overall channel characteristics.

Let $\mathbf{h}_k$, a $M \times 1$ vector, be the time sample vector of $h(t - kT)$, where the value of M will be considered later. Then (4) is equivalent to the vector equation

$$\mathbf{S} = \sum_{k=1}^{N} b_k \mathbf{h}_k \tag{7}$$

The N vectors $\mathbf{h}_k$, $k = 1, \cdots, N$, are linearly independent since the impulse responses $h(t - kT)$ are. Hence, the N vectors $\mathbf{h}_k$, $k = 1, \cdots, N$, generate a real Euclidean vector space $\mathcal{E}_N$ of N dimensions. If the precoder were not used, we would have $b_k = a_k$ and $\mathbf{S} = \sum_{k=1}^{N} a_k \mathbf{h}_k$, and the information symbols a_k would be transmitted as coordinates of the basis vectors $\mathbf{h}_k$ of $\mathcal{E}_N$. It is well known that the basis can be changed by a linear transformation. A precoder can be used for this purpose so that a suitable set of basis vectors can be chosen for each transmission rate of a multi-speed system based on considerations such as signal-to-noise ratio and the effect of timing error.

Define

$$\mathbf{A} = \begin{bmatrix} a_1 \\ \vdots \\ a_N \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} b_1 \\ \vdots \\ b_N \end{bmatrix}, \quad \mathbf{H}' = \begin{bmatrix} \mathbf{h}_1' \\ \vdots \\ \mathbf{h}_N' \end{bmatrix}, \quad \mathbf{V}' = \begin{bmatrix} \mathbf{V}_1' \\ \vdots \\ \mathbf{V}_N' \end{bmatrix}, \tag{8}$$

where $\mathbf{V}$ represents a set of basis vectors for $\mathcal{E}_N$ and the prime notation represents transpose. Since $\mathbf{h}_k$, $k = 1, \cdots, N$, generate $\mathcal{E}_N$, $\mathbf{V}$ is related to $\mathbf{H}$ by

$$\mathbf{V} = \mathbf{H\Lambda}, \tag{9}$$

where $\mathbf{\Lambda} = [\lambda_{ij}]$ is an $N \times N$ nonsingular matrix. If a_k is transmitted as a coordinate of $\mathbf{V}_k$, then

$$\mathbf{S} = \sum_{k=1}^{N} a_k \mathbf{V}_k = \mathbf{VA} = \mathbf{H\Lambda A}. \tag{10}$$

But, from (7)

$$\mathbf{S} = \mathbf{HB}. \tag{11}$$

From (10) and (11), the precoder structure is

$$\mathbf{B} = \mathbf{\Lambda A}. \tag{12}$$

Since the noise statistics and the statistics of the customer's data are usually unavailable, we choose here not to carry out a usual optimization study on the choice of **V** using such statistics. In the sequel, **V** is chosen to be a set of orthonormal basis vectors. This enables the precoding signaling scheme (Scheme II) to meet the following requirements:

(*i*) The performance is optimum in the same sense as the popular Scheme I described in Section I.

(*ii*) The overall channel characteristics are controlled by the precoding matrix $\mathbf{\Lambda}$ and hence by resistive elements. (In Scheme I the overall channel characteristics are controlled by the transmitting and receiving filters.)

These requirements are met with a simple receiver structure. The noisy signal at the input of the receiver is

$$\mathbf{X} = \mathbf{S} + \mathbf{N} = \sum_{k=1}^{N} a_k \mathbf{V}_k + \mathbf{N}, \tag{13}$$

where **N** is the noise vector. A correlator at the receiver computes the decision statistics $\mathbf{X}'\mathbf{V}_1, \mathbf{X}'\mathbf{V}_2, \cdots, \mathbf{X}'\mathbf{V}_N$. Since $\mathbf{V}_1, \mathbf{V}_2, \cdots, \mathbf{V}_N$ are orthonormal, we have

$$\mathbf{X}'\mathbf{V}_k = a_k + \mathbf{N}'\mathbf{V}_k\,. \tag{14}$$

Because of orthonormality the decision statistic $\mathbf{X}'\mathbf{V}_k$ depends only on a_k and there is no intersymbol interference. A decision on the symbol a_k can be made from the decision statistic $\mathbf{X}'\mathbf{V}_k$ by a simple, standard decision rule.

A basic difference between Schemes I and II is that in I the signals $f(t - kT_0)$ are time translates of each other, but in II the orthogonal signals $\mathbf{V}_k$ are not necessarily time translates. A difference in operation between the two schemes is seen in the second requirement. In Scheme I the overall channel characteristics are controlled by the transmitting and receiving filters. But, in Scheme II, they are controlled by the precoding matrix. To illustrate this and also for use in Section IV, we derive the impulse responses of Scheme II. As shown in Fig. 3, the correlator can be implemented with a tapped delay line and N sets of attenuators. Only the jth set of attenuators is shown. The attenuation ratios $V_{j1}, \cdots, V_{jM}$ shown are the values of the elements of $\mathbf{V}_j$, and the decision statistic $\mathbf{X}'\mathbf{V}_j$ is obtained by sampling the output of the jth summing circuit. For analytical purposes, the tapped delay

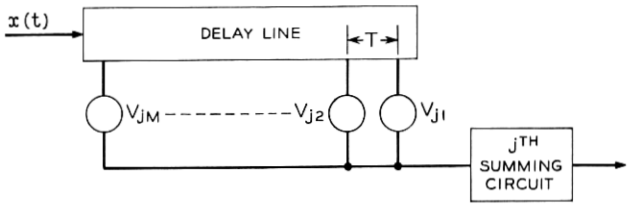

Fig. 3 — Diagram defining h_{ij} (t).

line, the jth set of attenuators, and the jth summing circuit together are equivalent to a matched filter having impulse response $V_j(t_0 - t)$, where t_0 is the sampling instant and $V_j(t)$ is a signal whose time sample vector is $\mathbf{V}_j$. Now define

$$h_{ij}(t) = \text{output of the } j\text{th summing circuit when } a_i = 1 \text{ is applied to the precoder.} \tag{15}$$

Since a_i is transmitted by the signal $\mathbf{V}_i$ or $V_i(t)$, we have

$$h_{ij}(t) = \int_{-\infty}^{\infty} V_j(t_0 + \tau - t)V_i(\tau)\, d\tau. \tag{16}$$

From (8) and (9)

$$\mathbf{V}_j = \sum_{k=1}^{N} \lambda_{jk}\mathbf{h}_k . \tag{17}$$

From (16) and (17)

$$h_{ij}(t) = \sum_{k=1}^{N} \sum_{l=1}^{N} \lambda_{jk}\lambda_{il} \int_{-\infty}^{\infty} h(t_0 + \tau - t - kT)h(\tau - lT)\, d\tau. \tag{18}$$

It is seen from (18) that, for a given transmission medium, $h_{ij}(t)$ is controlled by the elements λ_{ij} of the precoding matrix. Since changing the precoding matrix requires only changing attenuation ratios in the precoder and the correlator, the overall channel characteristics are controlled by resistive elements.

III. PRECODING FOR MULTIPLE-SPEED TRANSMISSION

The transmission of a single block has been considered in Section II. Now consider the transmission of an infinite sequence of symbols. In Scheme II, a symbol sequence is divided into blocks with N symbols in each block. If the vectors $\mathbf{h}_1, \cdots, \mathbf{h}_N$ are $M \times 1$ as assumed in Section II, the blocks can be transmitted sequentially at MT seconds intervals without interference between each other and the data rate is

$$R = \frac{N}{M} R_{\max} \quad \text{bauds,} \tag{19}$$

where $R_{\max}$ is the Nyquist rate.

Theoretically there is no limit on the block length N; however, we shall restrict N to be small number such as 3 so that the precoder and the correlator can be easily implemented. The parameter M must be restricted accordingly so that R [see (19)] can be a commonly used data rate such as 3/4 of the Nyquist rate. These requirements are satisfied by using the popular partial response channels.[2,3]

Table I of Ref. 3 illustrated five classes of partial response channels. From the table, it is clear that if $h(t)$ is in Class 1, then a set of sampling instants can be chosen (sampling time error will be considered later) such that $h(t - T), \cdots, h(t - NT)$ are simultaneously zero at all except $N + 1$ adjacent sampling points. This means that the vectors $\mathbf{h}_1, \cdots, \mathbf{h}_N$ are each $(N + 1) \times 1$ so that

$$\begin{aligned} M &= N + 1 \\ R &= \frac{N}{N+1} R_{\max} \quad \text{bauds.} \end{aligned} \tag{20}$$

If $h(t)$ is in Class 2, or 3, or 4, sampling instants can be chosen such that $M = N + 2$. The rule can be easily extended to other classes.

Consider now multiple-speed operation. As will be shown it is desirable to change the overall channel characteristics when changing the data rate. To make these changes, it is necessary to change the data format; however, it is desired that the system not be altered significantly otherwise (such as changing the filters, the equalization, the signaling interval, the receiver sampling time, etc.).

The scheme developed allows the data rate and overall channel characteristics to be changed simultaneously by changing only the data format and some resistive elements. When the system operates as above, the data rate is $(N/M)R_{\max}$ bauds and the sequence of symbols

$$a_1 a_2 \cdots a_N a_{N+1} \cdots$$

is transmitted. If the particular channel is noisy, one may wish to reduce the baud rate so that signal energy per baud can be increased to combat noise (an adaptive technique). The data rate can be changed to

$$R = \frac{r}{M} R_{\max} \quad \text{bauds,} \tag{21}$$

where r can be any integer from 1 to N, by inserting $N - r$ zero digits into each block as follows

$$a_1 \cdots a_r \quad 0 \cdots 0 \quad a_{r+1} \cdots a_{2r} \quad 0 \cdots 0 \quad a_{2r+1} \cdots$$

and transmitting this sequence instead of the original symbol sequence. The r information symbols in each block are transmitted to the first r summing circuits of the correlator at the receiver, while the $N - r$ zero digits in each block are transmitted to the other summing circuits. For convenience, let us refer to the transmission path from the precoder to the jth summing circuit as the jth subchannel. Since there is no information transmission through the last $N - r$ subchannels, it is no longer necessary to consider their performances. The precoding matrix $\mathbf{\Lambda}$ can be changed to improve the performance of the first r subchannels (such as reducing the effect of timing error). This can be done by changing the resistive elements in the precoder and correlator.

To summarize, the multiple-speed transmission scheme has the following properties:

(*i*) Changing data rate and overall channel characteristics requires only changing the data format and some resistive elements. There is no change to the filters, the equalization, the signaling interval T, or the receiver sampling time.

(*ii*) With correct timing and the use of orthonormal signals, signal-to-noise ratio is maximized at each data rate for band-limited white noise under the constraints of fixed line signal power and no intersymbol interference.

(*iii*) By using partial response channels, commonly used data rates are easily obtained, using a physically realizable precoder and correlator.

The discussions so far are general. To show how the method can be applied, and, more important, to demonstrate the advantage of changing the overall channel characteristics when changing the data rate, we consider in detail a two-speed transmission scheme in Section IV.

IV. TWO-SPEED TRANSMISSION AND EYE PATTERNS

Consider the following problem: The transmission medium is equalized for transmission at half the Nyquist rate and

$$\begin{aligned} H(j\omega) &= \text{square root of full-cosine rolloff characteristic} \\ &= k \cos \frac{\pi f}{2f_c}, \qquad |\omega| \leqq 2\pi f_c \\ &= 0, \qquad \text{otherwise,} \end{aligned} \tag{22}$$

where k is a gain factor and f_c is the bandwidth. It is recognized that, if Scheme I is used, the system is simply the popular full-cosine rolloff system transmitting at half the Nyquist rate.

The channel can be utilized more efficiently if the transmission rate can be changed according to the noise level. To compromise between efficiency and equipment complexity we choose to consider here two-speed transmission and two common data rates, $\frac{1}{2}$ and $\frac{3}{4}$ of the Nyquist rate.

We consider in detail how Scheme II can be used for this purpose. Note that $H(j\omega)$ in (22) is the Class 1 partial response system function. Therefore, from (20) and (21)

$$R = \frac{r}{N+1} R_{\max} \quad \text{bauds,} \tag{23}$$

where r can be any integer from 1 to N. To obtain $\frac{1}{2}\, R_{\max}$ and $\frac{3}{4}\, R_{\max}$ from (23), N can be 3, 7, etc. We choose $N = 3$ so that the precoder and correlator can be easily implemented.

To obtain the higher data rate, the sequence of information symbols is divided into blocks with three digits in each block, where the nth block contains the symbols a_{3n+1}, a_{3n+2}, and a_{3n+3}. The blocks are applied to the precoder sequentially at $4T$ intervals. The precoder converts the symbols a_{3n+1}, a_{3n+2}, and a_{3n+3} in the nth block into numbers b_{3n+1}, b_{3n+2}, and b_{3n+3} and transmits b_{3n+i} at $t = (4n + i)T$.

Consider the block containing a_1, a_2, and a_3. The precoder converts a_1, a_2, and a_3 into b_1, b_2, and b_3, and transmits b_1, b_2, and b_3 sequentially at $t = T, 2T, 3T$. This produces, as discussed in the previous section, a signal at the receiver input as

$$\mathbf{X} = b_1\mathbf{h}_1 + b_2\mathbf{h}_2 + b_3\mathbf{h}_3 + \mathbf{N} \tag{24}$$

where the time sample vectors $\mathbf{h}_1$, $\mathbf{h}_2$, and $\mathbf{h}_3$ can be written as (omitting a gain factor and the common zero samples)

$$\mathbf{h}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \qquad \mathbf{h}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \qquad \mathbf{h}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}. \tag{25}$$

Equation (25) shows that if sampling time is correct (timing error will be considered later), $\mathbf{h}_1$, $\mathbf{h}_2$, and $\mathbf{h}_3$ are limited to a $4T$ time interval.

Since each block is transmitted in a $4T$ time interval, there is no interference between adjacent blocks.

The vectors $\mathbf{h}_1$, $\mathbf{h}_2$, and $\mathbf{h}_3$ generate a three-dimensional real Euclidean vector space $\mathcal{E}_3$. Let

$$\mathbf{V}_1 = \begin{bmatrix} V_{11} \\ V_{12} \\ V_{13} \\ V_{14} \end{bmatrix}, \qquad \mathbf{V}_2 = \begin{bmatrix} V_{21} \\ V_{22} \\ V_{23} \\ V_{24} \end{bmatrix}, \qquad \mathbf{V}_3 = \begin{bmatrix} V_{31} \\ V_{32} \\ V_{33} \\ V_{34} \end{bmatrix} \tag{26}$$

be a set of orthonormal basis vectors for $\mathcal{E}_3$ and let a_1, a_2, and a_3 be transmitted as coordinates of $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_3$, respectively. Then the signal $\mathbf{X}$ at the input of the receiver must also be

$$\mathbf{X} = a_1\mathbf{V}_1 + a_2\mathbf{V}_2 + a_3\mathbf{V}_3 + \mathbf{N}. \tag{27}$$

The precoder structure then is

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \lambda_{11} & \lambda_{21} & \lambda_{31} \\ \lambda_{12} & \lambda_{22} & \lambda_{32} \\ \lambda_{13} & \lambda_{23} & \lambda_{33} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}, \tag{28}$$

where the λ_{ij}'s can be easily determined from (24), (25), (26), and (27). This precoder structure can be easily realized (Fig. 4).

The correlator at the receiver which computes the decision statistics $\mathbf{X}'\mathbf{V}_1$, $\mathbf{X}'\mathbf{V}_2$, and $\mathbf{X}'\mathbf{V}_3$ can also be easily realized (Fig. 3, $j = 1, 2, 3$; $M = 4$).

It is clear from Figs. 3 and 4 that the precoding matrix can be changed by simply changing the resistive elements (the attenuators) in the precoder and the correlator.

The transmission rate is $3/4\ R_{\max}$ when the system operates as above. To change the transmission rate to $1/2\ R_{\max}$, zero digits are inserted into the original data sequence as follows

$$\cdots\ a_1\ a_2\ 0\ a_3\ a_4\ 0\ a_5\ a_6\ 0\ \cdots,$$

and this new sequence is transmitted instead of the original sequence. Making use of the reduced baud rate to improve system performance, the overall channel characteristic is adjusted simultaneously by changing the precoding matrix. This is the subject of the following section.

4.1 *Timing Error and Eye Opening*

So far we have not specified which set of orthonormal basis vectors should be used. This is because with perfect timing the system meets

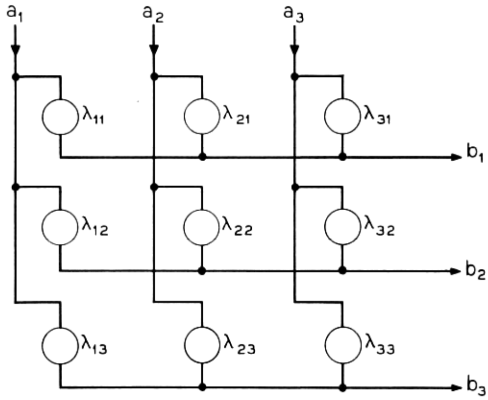

Fig. 4 — Precoder for two-speed transmission where λ_{ij}, $i, j = 1, 2, 3$, are attenuators.

the signal-to-noise ratio criterion in Section III regardless of which set of orthonormal basis vectors is chosen.

However, in practice, it is impossible to achieve zero sampling time error. In general, the receiver will sample the summing circuit outputs at $t = t_0 + \delta$ instead of the correct time t_0, where δ is a random timing error. Then the system's performance depends on the choice of $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_3$, i.e., depends on the choice of $\mathbf{\Lambda}$. To determine which $\mathbf{\Lambda}$ should be used, it is necessary to specify the type of transmission and choose a performance criterion accordingly.

In the sequel, we consider digital data transmission. Eye opening is adopted as the criterion since it is a widely accepted, practical one[4] (although considering eye openings in the presence of timing error leads to a difficult nonlinear mathematical problem).

Let $r_i(t)$ be the output of the ith summing circuit when an infinite sequence of digits is transmitted at the higher data rate $3/4\ R_{\max}$. Then

$$r_i(t) = \sum_{n=-\infty}^{\infty} a_{3n+i} h_{ii}(t - 4nT) + \sum_{j \neq i} \sum_{n=-\infty}^{\infty} a_{3n+j} h_{ji}(t - 4nT), \quad (29)$$

where $h_{ij}(t)$, as defined in (15), is the output of the jth summing circuit when $a_i = 1$ is transmitted alone. From (18) and (22) it can be shown that

$$\begin{aligned} h_{ij}(t) = {} & [\lambda_{i1}\lambda_{j1} + \lambda_{i2}\lambda_{j2} + \lambda_{i3}\lambda_{j3}]I(t) + [\lambda_{i2}\lambda_{j1} + \lambda_{i3}\lambda_{j2}]I(t - T) \\ & + [\lambda_{i1}\lambda_{j2} + \lambda_{i2}\lambda_{j3}]I(t + T) \\ & + \lambda_{i3}\lambda_{j1}I(t - 2T) + \lambda_{i1}\lambda_{j3}I(t + 2T), \end{aligned} \quad (30)$$

where

$$I(t) = \frac{1}{f_c^2} \frac{\sin 2\pi f_c(t - t_0)}{2\pi(t - t_0)[1 - 4f_c^2(t - t_0)^2]}. \tag{31}$$

To evaluate eye opening of $r_i(t)$ at $t_0 + \delta$, we assume that the information digits $\{a_i\}$ are binary and that each can be $\frac{1}{2}$ or $-\frac{1}{2}$ (so that full eye opening = 1). Then

$$\begin{aligned} E_i(\delta) &= \text{Eye opening of } r_i(t) \text{ at } t_0 + \delta \\ &= | h_{ii}(t_0 + \delta) | - \sum_{n=\pm 1}^{\pm\infty} | h_{ii}(t_0 + \delta - 4nT) | \\ &\quad - \sum_{j \neq i} \sum_{n=-\infty}^{\infty} | h_{ji}(t_0 + \delta - 4nT) | \\ &\qquad\qquad i, j = 1, 2, 3. \end{aligned} \tag{32}$$

Similarly, let $r_i'(t)$ be the output of the ith summing circuit when an infinite sequence of digits is transmitted at the lower data rate $1/2\ R_{\max}$. Since zeros are inserted and no information digit is received at the third summing circuit, we need to consider only the eye openings $E_1'(\delta)$ and $E_2'(\delta)$ of $r_1'(t)$ and $r_2'(t)$, respectively.

4.2 *Selection of Precoding Matrix*

It is seen that at the higher data rate, we must consider simultaneously $E_1(\delta)$, $E_2(\delta)$, and $E_3(\delta)$, while at the lower data rate we need only to consider $E_1'(\delta)$ and $E_2'(\delta)$. This suggests that a different precoding matrix should be selected for each data rate.

The steps in selection of the precoding matrix are lengthy and are outlined in the Appendix. The results are summarized here.

The precoding matrix selected for the higher data rate is

$$\begin{aligned} &\lambda_{11} = 0.21, \quad &&\lambda_{12} = 0.62, \quad &&\lambda_{13} = -0.5 \\ &\lambda_{21} = -0.68, \quad &&\lambda_{22} = 0.48, \quad &&\lambda_{23} = -0.68 \\ &\lambda_{31} = -0.5, \quad &&\lambda_{32} = 0.62, \quad &&\lambda_{33} = 0.21. \end{aligned} \tag{33}$$

Eye openings obtained with this precoding matrix are given in Table I for $\delta = 0, \pm 0.1T, \pm 0.2T, \pm 0.3T$ (it is a reasonable expectation that the timing error δ will amount to no more than $\pm 0.2T$). Also given in Table I is the eye opening $E(\delta)$ of the popular "raised cosine" rolloff

TABLE I—COMPARISON OF EYE OPENINGS

Timing Error δ	$E_1(\delta)$	$E_2(\delta)$	$E_3(\delta)$	$E(\delta)$
−0.3T	0.312	0.418	0.351	0.312
−0.2T	0.559	0.625	0.575	0.551
−0.1T	0.790	0.821	0.793	0.783
0	1.000	1.000	1.000	1.000
0.1T	0.793	0.821	0.790	0.783
0.2T	0.575	0.625	0.559	0.551
0.3T	0.351	0.418	0.312	0.312

system[5] which transmits at the same data rate 3/4 R_{max} (i.e., which utilizes a 33.3 percent rolloff band). A glance shows that the eye openings $E_1(\delta)$, $E_2(\delta)$, and $E_3(\delta)$ are equal to or larger than the eye opening $E(\delta)$ of the conventional system.

The precoding matrix selected for the lower data rate is

$$\lambda_{11} = \frac{1}{\sqrt{2}}, \qquad \lambda_{12} = 0, \qquad \lambda_{13} = 0$$

$$\lambda_{21} = 0, \qquad \lambda_{22} = 0, \qquad \lambda_{23} = \frac{1}{\sqrt{2}},$$

where λ_{31}, λ_{32}, and λ_{33} can be arbitrary since no information digit is transmitted through the third subchannel. With this precoding matrix, the system is identical with the popular "full cosine" rolloff system at the lower data rate, and the eye openings $E_1'(\delta)$ and $E_2'(\delta)$ are both 1.00, 0.955, 0.896, and 0.823, respectively, for δ equal to 0, $\pm 0.1T$, $\pm 0.2T$, and $\pm 0.3T$. These eye openings are much larger than $E_1(\delta)$ and $E_2(\delta)$ in Table I. This clearly demonstrates the advantage of changing the precoding matrix when changing the transmission rate.

V. CONCLUSIONS

A precoding scheme is presented for multiple-speed digital or analog data transmission. The scheme has the following properties

(*i*) Changing data rate and overall channel characteristics requires only changing the data format and some resistive elements. There is no change to the filters, the equalization, the transmitter signaling interval, or the receiver sampling time.

(*ii*) With correct timing and the use of orthonormal signals, the signal-to-noise ratio is maximized at each data rate for band-limited white noise under the constraints of fixed line signal power and no intersymbol interference.

(*iii*) By using partial response channels, a number of commonly used data rates are easily obtained using a physically realizable precoder and correlator.

Timing error is considered in a two-speed transmission scheme. Eye openings are used as the criterion in selecting the precoding matrix. Eye openings obtained are equal to or larger than those of two conventional schemes transmitting at the same data rates. The study clearly demonstrates the advantage of changing the overall channel characteristics when changing the data rate.

APPENDIX

Selection of Precoding Matrix

As can be seen from (32), (30), and (31), the problem of finding a precoding matrix for maximizing the eye openings in some joint sense over a certain range of the random variable δ is nonlinear and mathematically intractable. In the following, we reduce the dimension and range of the precoding-matrix space $\mathcal{S} = \{\Lambda\}$ to a minimum by using constraints and properties of $\mathcal{S}$, then derive a guide for searching the reduced space. Eye openings are obtained equal to or larger than those of two conventional schemes transmitting at the same data rates.

Consider the higher data rate. The eye openings $E_1(\delta)$, $E_2(\delta)$, and $E_3(\delta)$ are determined by the nine parameters λ_{ij}, $i, j = 1, 2, 3$. We have from orthogonality of $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_3$

$$h_{ij}(t_0) = 0, \qquad i, j = 1, 2, 3; \qquad i \neq j. \tag{34}$$

Define for $i = 1, 2, 3$

$$W_i = \frac{\lambda_{i1}}{\lambda_{i2}} + \frac{1}{2}; \qquad C_i = \frac{\lambda_{i3}}{\lambda_{i2}} + \frac{1}{2}. \tag{35}$$

It can be shown from (30), (31), and (35) that (34) is equivalent to the constraints

$$C_iC_j = -W_iW_j - \tfrac{1}{2}, \qquad i, j = 1, 2, 3; \qquad i \neq j. \tag{36}$$

Equation (36) is satisfied if and only if one of the following conditions holds

$$(i) \qquad W_1W_2 \leqq -\tfrac{1}{2}, \qquad W_2W_3 \leqq -\tfrac{1}{2}, \qquad W_3W_1 \leqq -\tfrac{1}{2} \tag{37}$$

$$(ii) \qquad W_1W_2 \leqq -\tfrac{1}{2}, \qquad W_2W_3 \geqq -\tfrac{1}{2}, \qquad W_3W_1 \geqq -\tfrac{1}{2} \tag{38}$$

$$(iii) \qquad W_1W_2 \geqq -\tfrac{1}{2}, \qquad W_2W_3 \geqq -\tfrac{1}{2}, \qquad W_3W_1 \leqq -\tfrac{1}{2} \tag{39}$$

$$(iv) \qquad W_1W_2 \geqq -\tfrac{1}{2}, \qquad W_2W_3 \leqq -\tfrac{1}{2}, \qquad W_3W_1 \geqq -\tfrac{1}{2}. \tag{40}$$

Each of the four conditions specifies a subspace of $\mathcal{S}$. Equation (37) corresponds to a null space because its requirements are conflicting. Equations (39) and (40) can be obtained from (38) by rotating the indexes of W; hence, for every point P in the subspace of (39) or (40), there is a point Q in the subspace of (38) such that P and Q produce eye openings differing only in indexes (for instance, P produces $E_1(\delta) = \alpha(\delta)$, $E_2(\delta) = \beta(\delta)$, and $E_3(\delta) = \gamma(\delta)$; Q produces $E_1(\delta) = \gamma(\delta)$, $E_2(\delta) = \alpha(\delta)$, and $E_3(\delta) = \beta(\delta)$). Since they are the same set of eye openings, we need only to cover the subspace of (38) in searching for $\mathbf{\Lambda}$.

The subspace of (38) can be further narrowed. It can be shown that (38) holds if and only if

$$W_1W_2 \leqq -\tfrac{1}{2}, \qquad W_2W_3 \geqq 0, \qquad -\tfrac{1}{2} \leqq W_3W_1 \leqq 0 \tag{41}$$

or

$$W_1W_2 \leqq -\tfrac{1}{2}, \qquad -\tfrac{1}{2} \leqq W_2W_3 \leqq 0, \qquad W_3W_1 \geqq 0. \tag{42}$$

Equation (42) can be obtained from (41) by exchanging W_1 and W_2. Thus, for the reason just cited, we need to search only the subspace of (41) instead of that of (38).

To further reduce $\mathcal{S}$, we divide the subspace of (41) into two subspaces

$$(i) \qquad W_1W_2 \leqq -\tfrac{1}{2}, \qquad W_2W_3 \geqq 0, \qquad -\tfrac{1}{2} \leqq W_3W_1 \leqq -\tfrac{1}{4} \tag{43}$$

$$(ii) \qquad W_1W_2 \leqq -\tfrac{1}{2}, \qquad W_2W_3 \geqq 0, \qquad -\tfrac{1}{4} \leqq W_3W_1 \leqq 0. \tag{44}$$

From (36), (44) can be written as

$$C_2C_3 \leqq -\tfrac{1}{2}, \qquad C_1C_2 \geqq 0, \qquad -\tfrac{1}{2} \leqq C_1C_3 \leqq -\tfrac{1}{4}. \tag{44a}$$

It can be shown from (32), (30), and (35) that simultaneously exchanging W_1 and C_1, W_2 and C_2, and W_3 and C_3 does not change the eye openings. From this it can be shown that for every point P in the subspace of (43), there is a point Q in the subspace of (44a) such that P and Q have eye openings differing only in indexes. Since (44a) is equivalent to (44), this implies that only the subspace of (44) needs to be searched instead of that of (41).

The space $\mathcal{S}$ to be searched has been reduced to only that of (44). The $W_2 - W_3$ plane is reduced to a narrow strip for all $W_1 \neq 0$. For instance, for $W_1 = -1$, W_3 is bounded between 0 and $\frac{1}{4}$, and W_2 and W_3 are bounded in the very narrow strip shown in Fig. 5.

Each point (W_1, W_2, W_3) in the subspace of (44) determines a precoding matrix through (36), (35), and the orthonormality condition $h_{ii}(t_0) = 1$.

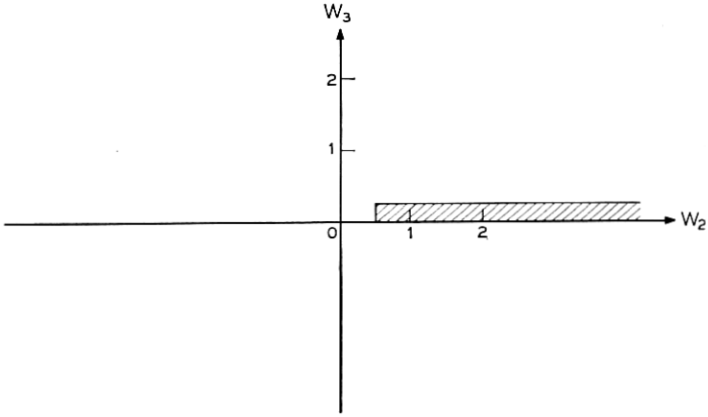

Fig. 5 — Region of W_2 and W_3 (shaded) when $W_1 = -1$.

Usually it is desirable that the three eye openings $E_1(\delta)$, $E_2(\delta)$, and $E_3(\delta)$ be approximately equal. It can be shown from (32), (30), and (31) that $E_1(\delta)$ and $E_3(\delta)$ are approximately equal if

$$W_1 = C_3 , \qquad W_2 = C_2 , \quad \text{and} \quad W_3 = C_1 . \tag{45}$$

Equation (45) defines the following region in the subspace of (44)

$$| W_1 | > \frac{1}{2} , \qquad W_2 = -\frac{2W_1}{4W_1^2 - 1} , \qquad W_3 = -\frac{1}{4W_1} . \tag{46}$$

$E_2(\delta)$ is larger than $E_1(\delta)$ and $E_3(\delta)$ at one extreme of the range $| W_1 | > \frac{1}{2}$, and is smaller at the other extreme. Therefore, in the region of (46), there are points at which $E_1(\delta)$, $E_2(\delta)$, and $E_3(\delta)$ are approximately equal. A simple search of this one-dimensional region gives one of such points as

$$W_1 = 0.84, \qquad W_2 = -0.92, \qquad W_3 = -0.3.$$

This point gives the precoding matrix in (33). Table I in Section IV shows that by using this precoding matrix for the higher data rate, the system has eye openings equal to or larger than the eye openings of a "raised cosine" rolloff system transmitting at the same data rate.

After the precoding matrix in (33) was obtained from the region of (46), the rest of the subspace of (44) was searched. About 5000 points were covered. It was found that no point had eye openings $E_1(\delta)$, $E_2(\delta)$, and $E_3(\delta)$ simultaneously larger than those in Table I.

A similar study for the lower data rate produced the result in Section IV.

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